Journal of Engineering Physics and Thermophysics, Vol. 71, No. 1, 1998

CALCULATION OF THE SURFACE DENSITY OF NUCLEATION SITES IN NUCLEATE BOILING OF A LIQUID

Yu. B. Zudin

UDC 539.216

On the basis of an approximate analysis of flow in a liquid macrofilm on a heating surface the characteristics of nucleate boiling (surface density of nucleation sites, effect of the thermal conductivity of the wall on the averaged heat transfer) are estimated.

Surface Density of Nucleation Sites. The surface density of nucleation sites n_{F} is one of the most important characteristics of nucleate boiling of liquids. At the present time, a rather large number of works are known in which the density of nucleation sites was counted by various methods [1-14]. However, this kind of measurement is reliable only in the regime of isolated bubbles, which is implemented at low heat loads or, as in [8], in forced flow of a subcooled liquid. Apparently, the density of nucleation sites in developed nucleate boiling was reliably determined only in [11-14]. In the majority of the above works the results of measurements were represented as relationships between the density of nucleation sites and the heat flux density $n_{\text{F}}(q)$ or the wall–liquid temperature drop $n_{\text{F}}(\Delta T)$. At the same time, as noted in [10], these relationships differ very strongly for different experimental conditions: from $n_{\text{F}} \sim \Delta T^2$ to $n_{\text{F}} \sim \Delta T^{25}$. In model developments [5, 7, 9, 14], the quantity n_{F} was used as an independent parameter in formulas for the heat-transfer coefficient in boiling of the form $\alpha \sim q^{m1} n_{\text{F}}^{m2}$. However, it is evident that the quantity n_{F} itself should be a function of the heat flux density q or the temperature drop ΔT. From general considerations it seems natural to relate the density of nucleation sites to the radius of the critical vapor nucleus R_*:

$$R_* = 2 \frac{\sigma T_{\text{s}}}{r \rho'' \Delta T}, \tag{1}$$

The first theoretical relation for the quantity n_{F} was established by Labuntsov in [15]. Proceeding from the physically justifiable assumption that among surface cavities of sizes on the order of R_* there are no predominant ones in the case of technically smooth surfaces, Labuntsov obtained the simplest relation for the density of nucleation sites:

$$n_{\text{F}} \approx \frac{1}{R_*^2} = \beta \left(\frac{r \rho'' \Delta T}{\sigma T_{\text{s}}} \right)^2 . \tag{2}$$

The constant β was estimated in [15] from experimental data obtained for boiling of water in an atmospheric environment: $\beta \sim 10^{-7}$–10^{-8}. According to [6], the value of β can reach 10^{-6}. A merit of formula (2) is that it is dimensionally correct, in contrast to the conventional form of representation [3]: $n_{\text{F}} \sim R_*^m$.

We note that in the majority of the experimental investigations [1-14] a much steeper dependence $n_{\text{F}}(\Delta T)$ than the quadratic dependence $n_{\text{F}} \sim \Delta T^2$ that follows from formula (2) was discovered. It seems, however, that the earlier attempts to correct formula (2) so as to have a larger exponent in the dependence $n_{\text{F}}(\Delta T)$ (see, for example, [16]) basically reduced to the physically unjustifiable purely empirical introduction of an additional linear scale.

Moscow Power Engineering Institute, Moscow, Russia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 71, No. 1, pp. 176-181, January, 1998. Original article submitted February 7, 1997.

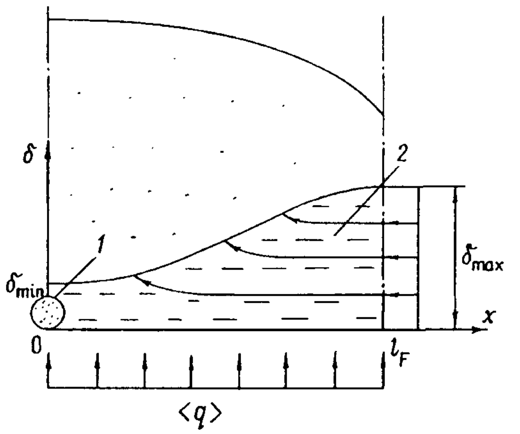

Fig. 1. Schematic of flow in a liquid macrofilm: 1) nucleate microroughness, 2) macrofilm.

It should be noted that in [15, 16] as well as in a more recent work by Labuntsov [17], expression (1) for the radius of the critical vapor nucleus R_* is obtained from a linear approximation of the saturation curve:

$$\Delta P = \frac{r\rho''\Delta T}{T_s}. \tag{3}$$

As shown by Yagov [18], for the region of low pressures and large temperature drops ($\Delta T \geq 0.1T_s$) a quadratic approximation of the saturation curve is more exact:

$$\Delta P = \frac{r^2\rho''\Delta T^2}{\tilde{R}T_s^3}. \tag{4}$$

Using formula (4) for the pressure drop ΔP along the saturation curve instead of formula (3) in the initial expression for the critical vapor nucleus

$$R_* = \frac{2\sigma}{\Delta P} \tag{5}$$

Yagov obtained

$$R_* = 2\frac{\sigma\tilde{R}T_s^3}{r^2\rho''\Delta T^2}. \tag{6}$$

Substitution of Eq. (6) into formula (2) leads to a dependence of the form $n_F \sim \Delta T^4$, which is valid for the region of low pressures. However, recent experimental investigations [11-14] clearly show that the noted steep dependence of the density of nucleation sites on the temperature drop ($n_F \sim \Delta T^6$ in [14]) is observed up to pressures close to critical ones. Presumably this indirectly indicates the existence of an additional (with respect to R_*) linear scale of the process of nucleate boiling. We made an attempt to determine such a scale on the basis of a consecutive physical model of the process.

Hydrodynamics of Inflow of a Liquid to "Dry Patches" on the Heating Surface in Boliling. An analysis of the hydrodynamics of evaporating menisci of a liquid film in the vicinity of "dry patches" on the heating surface provides the basis for the physical model of nucleate boiling developed by Yagov [18, 19]. In the present work we carry out an approximate analysis of inflow of a liquid from a macrofilm, located under vapor clusters on the heating surface, to the periphery of a dry patch (Fig. 1). It is assumed in this case that reduction in the thickness of the liquid film due to evaporation occurs until the film reaches a certain minimum size δ_{min}, comparable with the critical vapor nucleus radius:

$$\delta_{\min} = \beta_1 R_* . \tag{7}$$

Physically this means that the liquid film must link with a dry patch on scales of some conditional nucleate microroughness. In a first approximation the film thickness in the vicinity of a dry patch will be assumed constant: $\delta \approx \delta_{\min} = \text{const}$. Then, to estimate the longitudinal pressure gradient in the film we may use the analytical solution obtained in [20] for flow of liquid in a gap of constant thickness with a constant rate of suction (liquid evaporation) W prescribed on the upper boundary of the gap:

$$\frac{dP}{dx} = 3 \frac{\mu W}{\delta_{\min}^3} x . \tag{8}$$

To ensure inflow of liquid to the conditional boundary of the dry patch $x = 0$ (at a constant pressure in the vapor phase) it is necessary that the capillary pressure on the phase interface decrease along the coordinate x similarly to the problem of evaporation of menisci of thin liquid films considered in [21]. This means that the pressure gradient in the macrofilm will be determined by the gradient of the curvature of the interphase surface:

$$\frac{dP}{dx} = - \sigma \frac{d^3\delta}{dx^3} , \tag{9}$$

where $\delta(x)$ is the current film thickness (Fig. 1). Having determined the rate of suction (evaporation) of liquid on the upper boundary of the film W from the heat balance, from Eqs. (8) and (9) we obtain for the meniscus profile a differential equation that is asymptotically exact for $x \to 0$:

$$\frac{d^3\delta}{dx^3} = - 3 \frac{\nu \lambda \Delta T}{\sigma \delta_{\min}^4 r} x . \tag{10}$$

Let us assign the following conditions of smooth conjugation of the macrofilm with the microroughness at $x = 0$:

$$x = 0 : \ \delta = \delta_{\min} ; \ \frac{d\delta}{dx} = 0 . \tag{11}$$

From Eqs. (10) and (11) we obtain an expression for the meniscus profile for $x \to 0$:

$$\delta = \delta_{\min} + \beta_2 x^2 - \frac{1}{8} \left(\frac{\nu \lambda \Delta T}{\sigma \delta_{\min}^4 r} \right) x^4 . \tag{12}$$

Now we assign the condition for full operation of the pressure gradient (at a certain length l) in convergence of profile (12) with the macrofilm of thickness $\delta_{\max}$ that exists on the heating surface in nucleate boiling [15, 17-19]:

$$x = l : \ \delta = \delta_{\max} ; \ \frac{d^2\delta}{dx^2} = 0 . \tag{13}$$

As a result, we determine the constant β_2 in Eq. (12) and obtain a relationship that will connect together the geometric parameters of the macrofilm, viz., the length l, the minimum thickness $\delta_{\min}$, and the maximum thickness $\delta_{\max}$:

$$l = \beta_3 \delta_{\min} \left(\frac{\delta_{\max} \sigma r}{\nu \lambda \Delta T} \right)^{1/4} , \tag{14}$$

where $\beta_3 \sim 1$. Equations (12) and (14) yield a correlation for the heat-flux density $\langle q \rangle$ averaged over the length of the meniscus:

$$\langle q \rangle = \frac{\pi}{2} \frac{\lambda \Delta T}{(\delta_{\min} \delta_{\max})^{1/2}} . \tag{15}$$

According to [15, 17-19], we take the thickness of the effective viscous sublayer under the vapor clusters as the maximum thickness of the macrofilm $\delta_{\max}$:

$$\delta_{\max} = \beta_4 \frac{\nu r \rho''}{\langle q \rangle} , \tag{16}$$

where $\beta_4 \sim 1$.

Relations (15) and (16) yield the well-known Labuntsov formula [15, 17] for heat transfer in nucleate boiling:

$$\langle q \rangle = \beta_5 \frac{\lambda^2 \Delta T^3}{\nu \sigma T_s} , \tag{17}$$

where $\beta_5 \sim 10^{-3}$. Besides relation (17) that connects the heat flux to the temperature drop, from Eqs. (14)-(17) we also obtain an expression for the length of the macrofilm l:

$$\frac{l}{R_*} = \beta_6 \frac{(\rho'' \nu T_s)^{1/4} (\sigma r)^{1/2}}{\lambda^{3/4} \Delta T} , \tag{18}$$

where $\beta_6 \sim 10^2$. Equating the length l (in order of magnitude) to the characteristic distance between adjacent boiling centers, we obtain the quantity n_F, i.e., the surface density of nucleation sites:

$$n_F = \frac{1}{l^2} = \beta_8 \frac{r (\lambda \rho'')^{3/2} \Delta T^4}{\sigma^3 \nu^{1/2} T_s^{5/2}} , \tag{19}$$

where $\beta_8 \sim 10^{-5}$.

Effect of the Thermal Conductivity of the Wall on the Averaged Heat Transfer. As shown in [22], in the case of a nonuniform distribution of the thermal conductivity over the heat-transfer surface:

$$\alpha (x) = \frac{\lambda}{\delta (x)} \tag{20}$$

the averaged heat-transfer coefficient, defined as the ratio of the surface-averaged values of the heat-flux density $\langle q(x) \rangle$ and the temperature drop $\langle \Delta T(x) \rangle$:

$$\alpha_m = \frac{\langle q (x) \rangle}{\langle \Delta T (x) \rangle} , \tag{21}$$

should generally depend on the thermophysical properties of the wall. In this case, the quantity α_m will be smaller than the surface-averaged value of the thermal conductivity $\langle \alpha(x) \rangle$:

$$\alpha_m \equiv \frac{\langle q (x) \rangle}{\langle \Delta T (x) \rangle} \le \langle \alpha (x) \rangle \equiv \left\langle \frac{q (x)}{\Delta T (x)} \right\rangle . \tag{22}$$

Here the symbol $\langle \ \rangle$ indicates averaging over the space scale of periodicity along the x axis, in the present case, over the characteristic distance between adjacent sites. According to [22-24], for the spatial nonuniformity of heat transfer prescribed by the macrofilm profile (Eq. (12)), the averaged heat-transfer coefficient α_m will be equal to

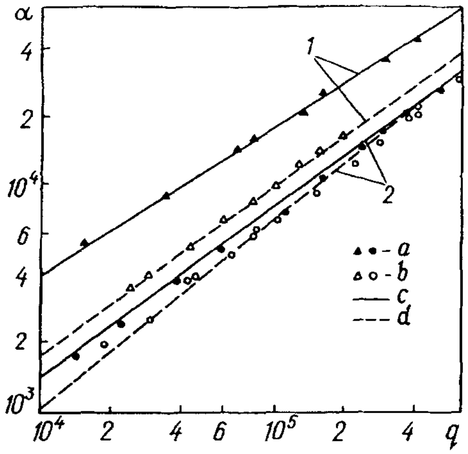

Fig. 2. Comparison of results of calculations by relation (23) with experimental data of [25] on heat-transfer in nucleate boiling of water on walls made of copper and stainless steel: 1) $P = 100$ kPa, 2) 20 (a wall made of copper (a), of stainless steel (b), boiling on a wall made of copper (c), of stainless steel (d). α, W/(m^2·K); q, W/m^2.

$$\varepsilon \equiv \frac{\alpha_{\mathrm{m}}}{\langle \alpha \rangle} = \left\{[A^2 + (1 + \varphi) A/\varphi^{1/2} + 1] - A\right\}^{-1}, \tag{23}$$

$$A \equiv \beta_9 \frac{\lambda^{5/4} T_{\mathrm{s}}^{1/4} \sigma^{1/2}}{\lambda_{\mathrm{w}} (\nu \rho'')^{3/4} r^{1/2}}, \tag{24}$$

$$\varphi \equiv \frac{\delta_{\min}}{\delta_{\max}} = \beta_{10} \frac{(\langle q \rangle \lambda \sigma T_{\mathrm{s}})^{2/3}}{\nu^{4/3} (r \rho'')^2}, \tag{25}$$

where λ_{w} is the thermal conductivity of the wall; β_9 and β_{10} are numerical constants.

For the limiting case of an infinitely conducting wall ($\lambda \to \infty$, $A \to \infty$) its influence on the averaged heat transfer degenerates ($\varepsilon \to 1$, $\alpha_{\mathrm{m}} \to \langle \alpha \rangle$). As follows from Eq. (25), the difference between the minimum $\delta_{\min}$ and maximum $\delta_{\max}$ thicknesses of the film decreases ($\varphi \to 1$) with decrease in pressure. Thus, the condition $\varphi \le 1$ determines the minimum possible value of the pressure at which the influence of the thermal conductivity of the wall λ_{w} on the averaged heat transfer degenerates ($\delta_{\min} = \delta_{\max}$; $\varphi = 1$; $\varepsilon = 1$; $\alpha_{\mathrm{m}} = \langle \alpha \rangle$). At a pressure lower than the given limiting one, the expounded computational scheme is inapplicable, since in this case we obtain $\varphi < 1$ ($\delta_{\max} < \delta_{\min}$). The indicated tendency of smoothing of the thermal effect of the wall on heat transfer in nucleate boiling of a liquid with decrease in pressure was obtained in the experimental investigation [25]. To fit our data to the results of [25], we had to increase the initial value of the constant β_6 in relation (18) for the length scale by a factor of ~50. Physically this apparently means that the quantity l specifies a certain longitudinal microscale on the heating surface that turns out to be smaller than the actual length scale by one-two orders of magnitude. Figure 2 presents a comparison of results of calculations by relation (23) with experimental data of [25] on heat transfer in nucleate boiling of water on walls made of copper and stainless steel for pressures of 100 and 20 kPa.

NOTATION

n_{F}, surface density of nucleation sites; ΔT, wall–liquid temperature drop; q, heat-flux density; R_*, radius of the critical vapor nucleus; $\tilde{R}$, individual gas constant; ΔP, pressure drop along the saturation curve; σ, coefficient of surface tension; T_{s}, saturation temperature; r, heat of phase transition; ρ'', vapor density; μ, ν, dynamic and kinematic viscosity of the liquid; λ, thermal conductivity of the liquid; x, longitudinal coordinate; δ, current thickness of the macrofilm; l, length of the macrofilm; $\delta_{\min}$ and $\delta_{\max}$, minimum and maximum values of the macrofilm thickness; W, rate of liquid suction; α_m, surface-averaged coefficient of heat transfer; $\langle\alpha\rangle$, thermal conductivity of the macrofilm; A, parameter of the thermal effect of the wall; φ, parameter of the spatial nonuniformity.

REFERENCES

1. C. Corty and A. S. Foust, Chem. Eng. Progr. Symp. Ser., **51**, No. 17, 1-12 (1955).
2. R. F. Gaertner, Proc. ASME, Ser. C, Heat Transfer, **87**, No. 1, 20-35 (1965).
3. M. Shukry and R. L. Judd, Proc. ASME, Ser. C, Heat Transfer, **100**, No. 4, 61-67 (1978).
4. R. F. Gaertner, Chem. Eng. Progr. Symp. Ser., **59**, No. 41, 52-61 (1963).
5. G. N. Danilova, Inzh.-Fiz. Zh., **11**, No. 3, 367-370 (1966).
6. Y. Heled and A. Orell, Int. J. Heat Mass Transf., **10**, No. 4, 553-554 (1967).
7. M. Sultan and R. L. Judd, Proc. ASME, Ser. C, Heat Transfer, **100**, No. 1, 59-66 (1978).
8. D. B. R. Kenning and M. V. H. Del Valle, Int. J. Heat Mass Transf., **24**, No. 6, 1025-1032 (1981).
9. G. Kosamustafaogullari and M. Ishii, Int. J. Heat Mass Transf., **26**, No. 9, 1377-1387 (1983).
10. J. E. Sgheiza and J. E. Myers, AIChE J., **31**, No. 10, 1605-1613 (1985).
11. G. Barthau, Int. J. Heat Mass Transf., **35**, No. 2, 271-278 (1992).
12. H. Schömann, Beitrag zum Einfluss der Heizflächenrauhigkeit auf den Wärmeübergang beim Blasensieden, Dissertation Dr.-Ing. Universität (GH). Paderborn (1994).
13. M. A. Lambert, Wärmeübergang beim Blasensieden reiner Stoffe im Bereich tiefer Siedetemperaturen, Dissertation Dr.-Ing. Universität (TH). Karlsruhe (1990).
14. A. D. Pinto, Wärmeübergang und Blasenbildung beim Sieden von Propan an einem geschmirgelten Kupferrohr in einem grossen Druckbereich, Dissertation Dr.-Ing. Universität (GH). Paderborn (1995).
15. D. A. Labuntsov, Izv. Akad. Nauk SSSR, Énerget. Trans., No. 1, 58-71 (1963).
16. V. M. Borishanskii and K. A. Zhokhov, Inzh.-Fiz. Zh., **15**, No. 5, 809-817 (1968).
17. D. A. Labuntsov, Teploénergetika, No. 9, 14-19 (1972).
18. V. V. Yagov, Approximate Theory of Heat Transfer and Crises of Nucleate Boiling of Liquids, Doctoral Dissertation, Moscow (1988).
19. V. V. Yagov, Teploénergetika, No. 2, 4-9 (1988).
20. V. V. Samsonov and D. A. Labuntsov, Trudy MÉI, **198**, 114-130 (1974).
21. F. I. Renk and P. S. Weiner, Jr., Proc. ASME, Ser. C, Heat Transfer, **101**, No. 1, 65-69, 70-74 (1979).
22. D. A. Labuntsov and Yu. B. Zudin, Processes of Heat Transfer with Periodic Intensity [in Russian], Moscow (1984).
23. Yu. B. Zudin, Teplofiz. Vys. Temp., **29**, No. 2, 395-398 (1991).
24. Yu. B. Zudin, Teplofiz. Vys. Temp., **29**, No. 6, 1134-1141 (1991).
25. A. K. Gorodov, Experimental Investigation of Boiling of Liquid in the Region of Low Pressures with the Boiling Surface Being Heated by a Circulating Liquid, Candidate's Dissertation, Moscow (1976).